

Static Optimization with MATLAB

Introduction

GSEFM Pre-Semester Course

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September 22-28, 2026 | Frankfurt

Hello!



- Luca Schmitz, 24 years old
- (Almost) fourth year PhD student
- Chair of Professor Wieland
- Member of the Monetary Policy Expert Panel of the European Parliament til 2028
- Research interest: Macroeconomic Risk & Forecasting, Monetary Economics, AI and the Macroeconomy
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Material

- Lecture Material, Codes, and (hopefully) recordings will be available:
 - will be available via **OLAT**
- Main references:
 - **de la Fuente, A. (2000).** *Mathematical methods and models for economists.* Cambridge University Press
 - **Chiang, A. C., & Wainwright, K. (2005).** *Fundamental methods of mathematical economics.* McGraw-Hill Education
 - **Werner, J. (2012).** Vorlesung über optimierung [Last visited on 2023/09/19]. <https://num.math.uni-goettingen.de/werner/optim.pdf>
 - Inspiration from the notes of previous years $\Rightarrow$ Special thanks to Phillip Schwedler, Saurabh Sharma, Teresa Schildmann, Ha Anh Le, and Hendrik Hegemann

Tentative Schedule

- Introduction (Tuesday, 22.09.)
- Unconstrained Optimization (Tuesday, 22.09.)
- MATLAB Unconstrained Optimization (Tuesday/Wednesday, 23.09.)
- Optimization with Equality Constraints (Wednesday, 23.09.)
- Optimization with Inequality Constraints (Thursday, 24.09.)
- Optimization with Mixed Constraints (Thursday, 24.09.)
- Special Cases: Optimization with Linear Constraints (Friday, 25.09.)
 - Linear Optimization
 - Quadratic Optimization
 - Linearly Constrained Nonlinear Optimization
- MATLAB Optimization with Constraints (Friday, 25.09.)
- Comparative Statics and Value Functions (Monday, 28.09.)

Design of the Course

- Morning Session: Theory
- Afternoon Session: Exercises & MATLAB
- Exercises with pen and paper
- No proofs, some definitions, otherwise just derivations and intuition

Questions?

Optimization Problem I

Definition (Static Optimization Problem)

An optimization problem consists of a set of feasible solutions M and an objective function $f : M \rightarrow \mathbb{R}$. The goal is to find the minimum value of $f(x)$ over all $x \in M$, which is expressed as:

$$\min_{x \in M} f(x)$$

- We constrain ourselves to the case that M is a subset of $\mathbb{R}^n$.

Optimization Problem II

- This subset is typically a solution set of finitely many equations and inequalities, i.e.

$$M = \{x \in \mathbb{R}^n \mid g(x) \leq 0, h(x) = 0\}.$$

- The restriction mappings $g : \mathbb{R}^n \rightarrow \mathbb{R}^l$ and $h : \mathbb{R}^n \rightarrow \mathbb{R}^m$ are called **constraint functions**.
- The $=$ and $\leq$ -relationship is understood component-wise.
- If necessary, an inequality can be multiplied by -1 so that we can assume that all inequality constraints are oriented in the same direction.
- Additionally, maximizing an objective function can be reduced to minimizing the objective function multiplied by -1 .

Optimization Problem III

- We will not discuss the case where M is a subset of an infinite-dimensional linear normed space (i.e. where we search for a function).
- In this case, we speak of an infinite optimization problem.
- Similarly, we will not address the important class of discrete (or combinatorial, integer) optimization problems, where the set M of feasible solutions is a finite set.

Optimization Problem IV

Definition (Solution)

- $x^* \in M$ is called a global minimum of the optimization problem if $f(x^*) \leq f(x)$ for all $x \in M$.
- $x^* \in M$ is called a strict global minimum of the optimization problem if $f(x^*) < f(x)$ for all $x \in M$.
- $x^* \in M$ is called a local minimum if there exists a neighborhood U^* of x^* such that $f(x^*) \leq f(x)$ for all $x \in M \cap U^*$.
- $x^* \in M$ is called a strict local minimum if there exists a neighborhood U^* of x^* such that $f(x^*) < f(x)$ for all $x \in M \cap U^*$.

Optimization Problem V

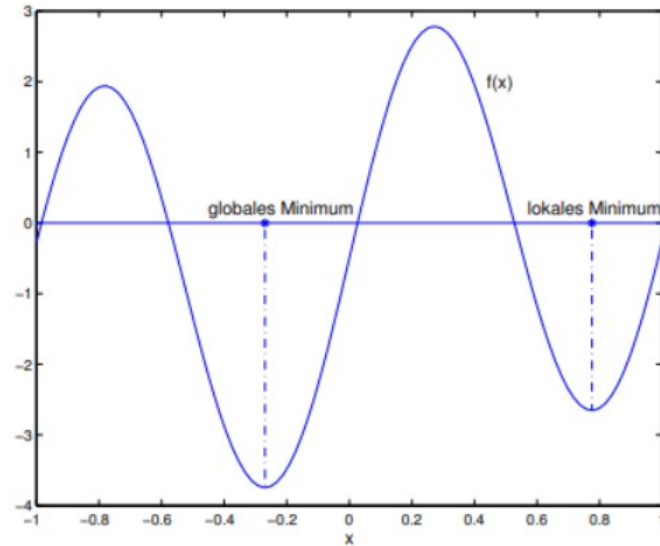


Figure: Global and local minimum of a function $f(\cdot)$ on $M = [-1, 1]$.

Unconstrained Optimization Problem

- If $M = \mathbb{R}^n$, we speak of an **unconstrained optimization problem**.
- To solve unconstrained optimization problems, MATLAB's Optimization Toolbox provides the function `fminunc`.
- In the simplest case, one can find a local minimum x of the function `fun` by calling `x=fminunc(fun,x0)`, starting from the initial value x_0 .

Linear Optimization

- If the objective function f is linear and the constraint mappings g and h are affine linear, we speak of a **linear optimization problem**.
- For example, $f(x) := c^T x$ with $c \in \mathbb{R}^n$, $g(x) := Ax - b$, and $h(x) := A_0 x - b_0$ with $A \in \mathbb{R}^{l \times n}$, $b \in \mathbb{R}^l$, $A_0 \in \mathbb{R}^{m \times n}$, and $b_0 \in \mathbb{R}^m$:

Minimize $c^T x$ on $M := \{x \in \mathbb{R}^n \mid Ax \leq b, A_0 x = b_0\}$.

- To solve linear optimization problems, MATLAB's Optimization Toolbox provides the function `linprog`. In the simplest case, one can call `x=linprog(c,A,b,A0,b0)`.

Quadratic Optimization

- If the objective function f is quadratic and the constraint mappings g and h are affine linear, we speak of a **quadratic optimization problem**.
- For example, $f(x) := \frac{1}{2}x^T Qx + c^T x$ with $Q \in \mathbb{R}^{n \times n}$ symmetric, $c \in \mathbb{R}^n$, $g(x) := Ax - b$, and $h(x) := A_0x - b_0$ with $A \in \mathbb{R}^{l \times n}$, $b \in \mathbb{R}^l$, $A_0 \in \mathbb{R}^{m \times n}$, and $b_0 \in \mathbb{R}^m$:

$$\text{Minimize } \frac{1}{2}x^T Qx + c^T x \text{ on } M := \{x \in \mathbb{R}^n \mid Ax \leq b, A_0x = b_0\}.$$

- To solve quadratic optimization problems, MATLAB's Optimization Toolbox provides the function `quadprog`. In the simplest case, one can call `x=quadprog(Q,c,A,b,A0,b0)`.

Linearly Constrained Nonlinear Optimization

- If the objective function f is nonlinear (and not quadratic) and the constraint mappings g and h are affine linear, we speak of a **linearly constrained nonlinear optimization problem**.
- For example, $f(x) : M \rightarrow \mathbb{R}$, with $g(x) := Ax - b$, and $h(x) := A_0x - b_0$ with $A \in \mathbb{R}^{l \times n}$, $b \in \mathbb{R}^l$, $A_0 \in \mathbb{R}^{m \times n}$, and $b_0 \in \mathbb{R}^m$:

Minimize $f(x)$ on $M := \{x \in \mathbb{R}^n \mid Ax \leq b, A_0x = b_0\}$.

- To solve linearly constrained nonlinear optimization problems, MATLAB's Optimization Toolbox provides the function `fmincon`. In the simplest case, one can call `x=fmincon(fun,x0,A,b,A0,b0)`.

Nonlinear Optimization

- If the objective function f or one of the constraint mappings g or h is not affine linear, we speak of a **nonlinear optimization problem**.
- We will only consider the case where f , g , and h are smooth, i.e., at least continuously differentiable.
- To solve nonlinear optimization problems, MATLAB's Optimization Toolbox provides the function `fmincon`.

References I

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